Faculty of Mathematics,
Bielefeld University

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Optimization and Dynamics

Summer semester 2015

Exercise sheet 2

Due 12pm, 24.04.2015

1. Define $f : \mathbb{N} \to \mathbb{N}$ by

$$f(x) = \begin{cases} 3x+1 & \text{if } x \text{ is odd} \\ \frac{x}{2} & \text{if } x \text{ is even} \,. \end{cases}$$

   (a) Is f injective? Is f surjective? (Explain why.)

   (b) Describe the orbits of x for $x = 1, 2, 3, 4$ and 5.

2. Let $f : \mathbb{R} \to \mathbb{R}$ be a continuous and invertible function and consider the discrete dynamical system defined by $x_{n+1} = f(x_n)$.

   (a) Prove that any periodic point of f must have minimal period $p \leq 2$.

   (b) Prove that f has no eventually periodic points.

3. Consider the dynamical system defined by the difference equation

$$x_{n+1} = a x_n (1 - x_n) \,,$$

   where $a = 2$.

   (a) Find and describe its fixed points.

   (b) Compare with the cases $1 < a < 2$ and $1 = a$.

4. Describe the fixed point(s) of the dynamical systems determined by the family of functions

$$f_c(x) = x^2 + c \,.$$

   (Hint: Consider the three cases $c < \frac{1}{4}$, $c = \frac{1}{4}$, and $c > \frac{1}{4}$ separately.)