Faculty of Mathematics,
Bielefeld University

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Optimization and Dynamics

Summer semester 2015

Exercise sheet 6

Due 12pm, 22.05.2015

1. Consider the dynamical systems defined by the following functions.

   (a) $f_a(x) = x + x^2 - a$

   (b) $g_a(x) = x + x^2 - a^2$

   In each case, by finding the fixed points and considering their stability in a nieghbourhood of $a = 0$, discuss the bifurcation that occurs at $a = 0$. Sketch the corresponding bifurcation diagrams.

2. Consider the family of dynamical systems defined by

$$f(x) = ax + x^4.$$

   Discuss the bifurcation that occurs at $a = 1$ and sketch the corresponding diagram.

3. In Theorem 4.6 (Saddle node bifurcation), we proved the existence of a function $h : K \to \mathbb{R}$ such that $h(\bar{x}) = \bar{a}$, $F(h(x), x) = x$ (for $x \in K$) and $h'(\bar{x}) = 0$. Show that

$$h''(\bar{x}) = -\frac{\frac{\partial^2 F}{\partial x^2}(\bar{a}, \bar{x})}{\frac{\partial F}{\partial a}(\bar{a}, \bar{x})}\,.$$

   Hint: Implicitly differentiate the expression $F(h(x), x) = x$ with respect to x. (Twice!)