Faculty of Mathematics,
Bielefeld University

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# Optimization and Dynamics

Summer semester 2015

## Exercise sheet 9

Due 12pm, 12.06.2015

1. Consider the dynamical system

$$\begin{pmatrix} x_{n+1} \\ y_{n+1} \end{pmatrix} = \begin{pmatrix} \frac{1}{2}\sin x_n + y_n \\ \frac{1}{2}y_n + x_n^2 \end{pmatrix}.$$

   Prove that $(0,0)$ is an attracting fixed point of the system.

2. Consider the discrete dynamical system given by the function $F : \mathbb{R}^2 \to \mathbb{R}^2$,

$$F\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \frac{1}{2}(x + x^3) \\ \frac{2y}{1+2x^2} \end{pmatrix}, \quad \text{for} \quad \begin{pmatrix} x \\ y \end{pmatrix} \in \mathbb{R}^2.$$

   (a) Determine the fixed points.

   (b) Show that all fixed points are saddle points.

   (c) Sketch the phase portrait of $F$.

3. Consider the logistic map $f_a : \mathbb{R} \to \mathbb{R}$, $f_a(x) = ax(1-x)$.

   (a) Let $a = 4$ and describe $O^+(\frac{1}{2})$, that is, the orbit of $x = \frac{1}{2}$ under $f_4$. Show that this orbit is unstable with respect to $[0,1]$.

   (b) Compare this with the case $a = 1$. That is, find the orbit $O^+(\frac{1}{2})$ under $f_1$ and discuss the stability of the orbit with respect to $[0,1]$.