Präsenzübungen zu
Vertiefung Elementare Zahlentheorie

WS 2010/2011,   Blatt 11

Präsenzaufgabe 41. (a) It is known that a primitive pythagorean triple (x, y, z) with even x has the form $(2uv, u^2 - v^2, u^2 + v^2)$ with $u > v > 0$, u and v relatively prime, u and v not both odd. Show that $\frac{y+z}{x} = \frac{u}{v}$.
(b) Check that $(28, 45, 53)$ is a primitive pythagorean triple and calculate the corresponding integers u and v.

Präsenzaufgabe 42. Let p be a prime and x, y, z integers. Show:
(a) $x^p + y^p = z^p \implies p \mid x + y - z$;
(b) $x^{p-1} + y^{p-1} = z^{p-1} \implies p \mid x$ or $p \mid y$ or $p \mid z$.

Präsenzaufgabe 43. Which of the integers $n = 150, 151, 152, 153, 154$ are sums of two squares? Find, if possible, a representation $n = x^2 + y^2$.

Präsenzaufgabe 44. Which of the integers $n = 100, 101, \ldots, 150$ are not sums of three squares?