Übungen zu Vertiefung Elementare Zahlentheorie

WS 2010/2011,   Blatt 3

Exercise 9. Let a and b be integers > 0 where

$$a = p_1^{n_1} p_2^{n_2} \cdots p_r^{n_r}$$

with prime numbers $p_1 < p_2 < \ldots < p_r$ and exponents $n_1 \geq 1, n_2 \geq 1, \ldots, n_r \geq 1$. Prove: $b \mid a$ if and only if

$$b = p_1^{m_1} p_2^{m_2} \cdots p_r^{m_r}$$

with exponents $0 \leq m_1 \leq n_1, 0 \leq m_2 \leq n_2, \ldots, 0 \leq m_r \leq n_r$.

Exercise 10. For any integer $a > 0$, $\tau(a)$ denotes the number of divisors > 0 of a. Determine $\tau(1024)$ and $\tau(5040)$.

Exercise 11. Prove for integers a, b, r, s with $r > 0$, $s > 0$:

$$r \mid s \text{ and } \frac{s}{r} \text{ odd} \quad \implies \quad a^r + b^r \mid a^s + b^s.$$

Exercise 12. Prove: If $2^N + 1$ (N an integer ≥ 1) is a prime number, then N is a power of 2, i.e., $N = 2^n$ for some integer $n \geq 0$.
(*Hint:* Exercise 11.)

Abgabe bis Freitag, 5.11.2010, 12:00 Uhr