

# Übungen zu Vertiefung Elementare Zahlentheorie

WS 2010/2011, Blatt 7

**Aufgabe 25.** Show for every prime number  $p$  and every  $0 \leq k \leq p - 1$ :

$$k!(p - 1 - k)! \equiv (-1)^{k+1} \pmod{p}.$$

**Aufgabe 26.** Determine the following values of Euler's  $\phi$ -function:

(a)  $\phi(36)$ ,  $\phi(360)$ ,  $\phi(3600)$ ;

(b)  $\phi(42)$ ,  $\phi(420)$ ,  $\phi(4200)$ .

**Aufgabe 27.** Show for  $n \geq 1$ :

(a)  $\phi(n) = n \iff n = 1$ ;

(b)  $\phi(n) = n/2 \iff n = 2^a$  with  $a \geq 1$ ;

(c)  $\phi(n) = n/3 \iff n = 2^a 3^b$  with  $a \geq 1, b \geq 1$ .

**Aufgabe 28.** (a) Show that there is no  $n \geq 1$  such that  $\phi(n) = 14$ .

(b) Determine all  $n \geq 1$  such that  $\phi(n) = 24$ .

**Abgabe bis Freitag, 3.12.2010, 12:00 Uhr**