Übungen zu Vertiefung Elementare Zahlentheorie

WS 2010/2011, Blatt 12

Aufgabe 45. Let p be a prime such that $p \equiv 3 \pmod 8$ and $(p-1)/2$ is a prime. Show that 2 is a primitive root modulo p. Find three examples of such primes p.

Aufgabe 46. Let p be a prime such that $p \equiv 7 \pmod 8$ and $(p-1)/2$ is a prime. Is $(p-1)/2$ a quadratic residue modulo p? Find three examples of such primes p.

Aufgabe 47. Let (x, y, z) be a primitive pythagorean triple. Show:

(a) If x is even, x is divisible by 4.

(b) Exactly one of x and y is divisible by 3; z is not divisible by 3.

(c) Exactly one of x, y and z is divisible by 5.

Aufgabe 48. (a) Find all pythagorean triples of the form $(x, x+y, x+2y)$ (the components should form an "arithmetic progression").

(b) Find all pythagorean triples of the form (x, xy, xy^2) (the components should form a "geometric progression").

Abgabe bis Freitag, 21.1.2011, 12:00 Uhr