Übungen zu Vertiefung Elementare Zahlentheorie

WS 2010/2011,   Blatt 13

Aufgabe 49. (a) Express all primes $p < 100$ with $p \equiv 1 \,(\mathrm{mod}\, 4)$ as sums of two squares.

(b) Express the integers $325, 442, 2880, 10048, 10049$ as sums of two squares.

Aufgabe 50. Which of the integers $n = 1965, 1966, 1967, 1968, 1969, 1970$ are sums of two squares? Find, if possible, a representation of the form $n = x^2 + y^2$.

Aufgabe 51. Let (x, y, z) be a primitive pythagorean triple. Show: Any prime divisor p of z satisfies $p \equiv 1 \,(\mathrm{mod}\, 4)$ (one has $z = u^2 + v^2$, where u and v are in particular relatively prime).

Aufgabe 52. Show:
(a) If $x^2 + y^2 + z^2$ is divisible by 4, then x, y, z are even;

(b) any integer of the form $8k + 7$ with $k \geq 0$ is not a sum of three squares;

(c) any integer of the form $4^a(8k + 7)$ with $a \geq 0, k \geq 0$ is not a sum of three squares. (*Hint*: use induction on a)

Abgabe bis Freitag, 28.1.2011, 12:00 Uhr