

## Corrections by Darij Grinberg

- page 18, Properties, (3): After "Explicitly", add ", if".
- page 21, proof of Theorem: In the proof of injectivity of  $\theta$ , you write: " $\text{tip}(r_{s'}) = w \text{tip}(r_s)$ ". This should be " $\text{tip}(r_{s'}) = \text{tip}(r_s) w$ " instead.
- page 22, proof of Proposition: Replace "length  $> 1$ " by "length  $\geq 1$ " on the last line of the proof.  
But more importantly, I think the proof of the proposition can be simplified: Assume that  $M$  is a finite-dimensional  $K\langle Q \rangle$ -module. Let  $N = \dim M$ . We shall show that  $p_M = 0$  for any path  $p$  of length  $> N$ . Indeed, let  $p = a_k a_{k-1} \cdots a_1$  be a path of length  $k > N$ . We must show that  $p_M = 0$ . In other words, we must show that  $p_m = 0$  for any  $m \in M$ . Thus, fix  $m \in M$ . We must prove  $p_m = 0$ . Assume the contrary; hence,  $p_m \neq 0$ . Set  $m_n = a_n a_{n-1} \cdots a_1 m$  for each  $0 \leq n \leq k$ . Then,  $m_k = p_m \neq 0$ .  
Now, consider the sequence of vector subspaces  
 $\langle m_0, m_1, \dots, m_k \rangle$ ,  
 $\langle m_1, m_2, \dots, m_k \rangle$ ,  
 $\dots$ ,  
 $\langle m_{k-1}, m_k \rangle$ ,  
 $\langle m_k \rangle$   
of  $M$ . Each of these subspaces contains the next one as a subset, and so their dimensions are weakly decreasing. Moreover, the dimension of the first one is  $\leq \dim M = N$ , whereas the dimension of the last one is  $1$  (since  $m_k \neq 0$ ). Thus, the dimensions appearing in these sequence are numbers between  $1$  and  $N$ . Consequently, two of these dimensions must be equal (since in total, the sequence contains  $k + 1 > k > N$  dimensions, but there are only  $N$  numbers between  $1$  and  $N$ ). In other words, there exist some  $i$  and  $j$  with  $i < j$  such that the subspaces  $\langle m_i, m_{i+1}, \dots, m_k \rangle$  and  $\langle m_j, m_{j+1}, \dots, m_k \rangle$  have the same dimension. Of course, these two subspaces must therefore be equal (since the latter is included in the former). Thus,  $m_i \in \langle m_j, m_{j+1}, \dots, m_k \rangle$ . Hence,  $m_i = x m_i$  where  $x$  is some linear combination of paths of length  $\geq 1$ . This rewrites as  $(1 - x) m_i = 0$ . This yields  $m_i = 0$ , since  $1 - x$  is invertible in the ring  $K\langle Q \rangle$ . Thus,  $p_m = 0$ , since  $p_m$  is a left multiple of  $m_i$ . This contradicts  $p_m \neq 0$ . This contradiction completes the proof.
- page 27, definitions of "overlap ambiguity" and "inclusion ambiguity": You should probably say that  $f$  means the word in question.
- page 31, proof of (the first) Lemma: On the last line of the proof, the  $\cong$  sign between  $L$  and  $N$  should probably be a  $\cap$  sign.
- page 32, Example, (ii): The displayed equation (which defines  $d/(dx)$ ) should not end with a period.
- page 33, proof: "a polynomial  $f = \sum r_i X^i$ " --> "a polynomial  $f = \sum r_i X^{i_1}$ ".
- page 33, proof: On the last line of the proof, " $\widehat{r}$ " should probably be defined (or replaced by " $r$ ").
- page 36, proof of Proposition: Replace " $[r, P] = \forall r$ " by " $[r, P] = 0 \forall r$ " (on the first line of the proof).

- page 42, definition of graded/homogeneous submodules: Replace " $n \in Z$ " (under the direct-sum sign) by " $n \in \mathbb{Z}$ ".
- page 45, first Definition: "if  $\theta : R \rightarrow A$  is a ring"  $\rightarrow$  "if  $\theta : R \rightarrow A$  is a ring homomorphism".
- page 45, Construction: "left Ore set" may be better off explicitly defined (you only introduced the "left Ore condition").  
More substantially: In " $(s, m) \sim (s', m)$ ", the second  $m$  should be an  $m'$ .
- page 58: Missing period after "Definition".